FLOW STABILITY IN A PLANE-PARALLEL CHANNEL WITH ONE PERMEABLE WALL

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We have calculated the characteristics of neutral flow stability in a plane-parallel channel with injection or suction through a permeable wall.

The stability of hydrodynamically developed symmetric flow in a plane-parallel channel with two permeable walls was examined in [1, 2]. In those papers the effect of injection and suction on the characteristics of neutral stability was investigated. Varapaev and Yagodkin [3] studied flow stability in a plane-parallel channel with injection through one wall and suction at the same rate through the other, so that the flow rate remained constant. In the present article we examine the flow stability in a channel with one permeable wall through which there is uniform injection or suction. This problem contains all the characteristics of both flows studied earlier: nonparallel flow and an asymmetric axial velocity profile.

1. The velocity distributions for uniform injection or suction along the length of the channel for hydrodynamically developed flow are found from the self-similar solution of the Navier–Stokes equations, and have the form [4]

$$u_x = (U_0 - Vx/h) f'(\eta),\ u_y = Vf(\eta).$$

The velocity function $f(\eta)$ satisfies the equation

$$f^{IV} + R(f'f'' - ff''') = 0. \qquad (1)$$

We write the boundary conditions for Eq. (1) for one-sided injection (suction):

$$f(-1) = f'(-1) = f'(1) = 0,\ f(1) = 2. \qquad (2)$$

Equation (1) with boundary conditions (2) was solved in [5] for injection. The axial velocity distributions over a cross section of the channel are shown in Fig. 1 for various

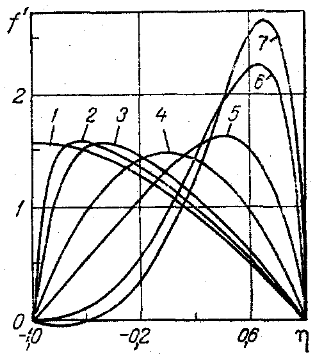

Fig. 1

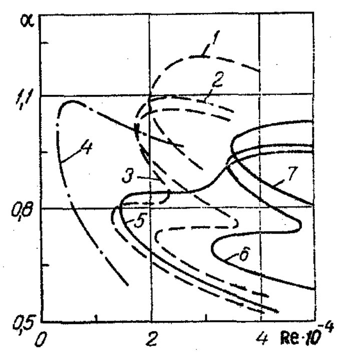

Fig. 2

Fig. 1. Profiles of axial velocity: 1) R = −∞; 2) −40; 3) −10; 4) 0; 5) 2; 6) 4; 7) 5.

Fig. 2. Neutral curves: 1) R = −3.25; 2) −2.0; 3) −1.75; 4) 0; 5) 0.85; 6) 1.0; 7) 1.25

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values of the parameter R. It is clear from the figure that as the injection rate is increased, the maximum velocity point is displaced toward the impermeable wall, and for suction it is displaced toward the permeable wall. A point of inflection appears on the velocity profile for suction; the maximum value of the relative velocity increases with increasing R, and for R = 3.28 the flow separates from the impermeable wall as a result of the positive pressure gradient.

For rapid injection the velocity profile, expect for a narrow region near the impermeable wall, is well described by the inviscid solution of Eq. (1) (omitting the first term):

$$f = 2\sin[\pi(1+\eta)/4]. \tag{3}$$

For rapid injection a boundary layer of thickness $0(R^{-1/2})$ is formed at the impermeable wall. By introducing the new variables $\xi = (\pi|R|/2)^{1/2}(1+\eta)$ and $F = (2|R|/\pi)^{1/2}f$ the solution of Eq. (1) is reduced to the solution of the flow problem near the frontal point of a cylinder [6]

$$F''' + FF'' - F'^2 + 1 = 0;\ F(0) = F'(0) = 0,\ F'(\infty) = 1. \tag{4}$$

Experiments [7] showed that the velocity distribution for rapid injection was actually close to the inviscid profile (3) over the whole cross section of the channel except for the boundary layer region near the impermeable wall where the experimental points are in good agreement with the solution of problem (4).

2. The stability characteristics of the flow under consideration can be calculated from the modified Orr—Sommerfeld equation [1]

$$\varphi^{IV} - 2\alpha^2\varphi'' + \alpha^4\varphi = i\alpha\,\mathrm{Re}\,[(f'-c)(\varphi''-\alpha^2\varphi) - f'''\varphi] + R\,[f(\varphi'''-\alpha^2\varphi') - f''\varphi']. \tag{5}$$

Here the half-width of the channel h is taken as the unit of length, and the mean velocity U in the cross section under consideration is taken as the unit of velocity. The boundary conditions for Eq. (5) are

$$\varphi(-1) = \varphi'(-1) = \varphi(1) = \varphi'(1) = 0. \tag{6}$$

The eigenvalues of Eq. (5) with boundary conditions (6) were determined by the differential pivotal method described in [8].

Figure 2 shows the neutral curves calculated for various injection and suction rates. It is clear from the figure that just as for flow with a constant transverse velocity [3], two local minima appear on the curve $\mathrm{Re}(\alpha)$ in a certain range of R values as a result of the asymmetry of the axial velocity profile. For injection the minimum which exists for large values of α corresponds to the critical point on the impermeable wall. For low injection rates ($|R| < 1.86$) the most dangerous perturbations are those arising at the permeable wall, and for $|R| > 1.86$ those at the impermeable wall. This accounts for the behavior of the relations between the critical values of the neutral stability parameters and the injection rate (Fig. 3a): the discontinuities in the graphs of $\alpha_*(R)$ and $c_*(R)$, the kink in the graph of $\mathrm{Re}_*(R)$. A comparison of the calculated dependence of Re_* on R with the corresponding curve for symmetrical injection [1] shows that in the present case the destabilizing effect of injection is considerably less, and is observed only for very low injection rates. As in two-sided injection, when $|R|$ is increased, the influx of mass begins to exert a stabilizing effect (in the sense of an increase in Re_*). However, in contrast with two-sided injection [2], the stability loss mechanism for rapid one-sided injection has a viscous character and is related to the development of perturbation at the impermeable wall.

For rapid injection, taking account of the fact that the most dangerous perturbations are those which develop at the impermeable wall, the critical parameters can be calculated from the solution of the flow stability problem near the frontal point of a cylinder. Changing from the variable η and f to ξ and F, and setting

$$\alpha_1 = \left(\frac{2}{\pi|R|}\right)^{1/2}\alpha,\ c_1 = \frac{2}{\pi}c,\ \mathrm{Re}_1 = \left(\frac{\pi}{2|R|}\right)^{1/2}\mathrm{Re},$$

transforms Eq. (5) with boundary conditions (6) into the form

$$\varphi^{IV} - 2\alpha_1^2\varphi'' + \alpha_1^4\varphi = i\alpha_1\mathrm{Re}_1[(F'-c_1)(\varphi''-\alpha_1^2\varphi) - F'''\varphi] - F(\varphi'''-\alpha_1^2\varphi') + F''\varphi', \tag{7}$$

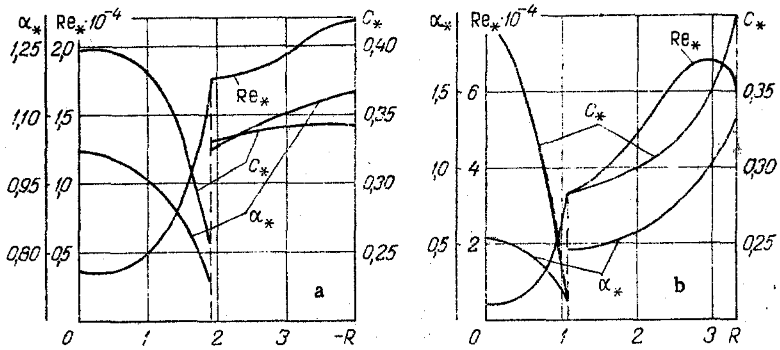

Fig. 3. Effect of a) injection and b) suction on critical parameters.

$$\varphi(0) = \varphi'(0) = \varphi(\infty) = \varphi'(\infty) = 0, \tag{7}$$

where the function $F(\xi)$ is determined from the solution of problem (4).

It follows from the solution of (7) that $Re_{1*} = 18{,}780$, $\alpha_{1*} = 0.25$, and $c_{1*} = 0.21$, from which we obtain the asymptotic relations for the critical parameters for rapid one-sided injection in a plane-parallel channel ($R \to -\infty$):

$$Re_* = 15000\,|R|^{1/2};\ \alpha_* = 0.31\,|R|^{1/2};\ c_* = 0.33.$$

It is clear from Fig. 3 that the effect of a low suction rate on flow stability is qualitively analogous to that of injection. For $R > 1.05$ the minimum of the $Re(\alpha)$ curve corresponding to larger values of α becomes controlling, and this leads to discontinuities in the graphs of the critical parameters. For $R > 3$ the critical Reynolds number begins to decrease with increasing suction rate, which is related to the decrease of the velocity gradient, flow separation, and the appearance of a zone of back flow at the impermeable wall.

NOTATION

x, y, longitudinal and transverse coordinates; u_x, u_y, longitudinal and transverse velocity components; h, half-width of channel; ν, kinematic viscosity; 2V, injection (suction) rate; U_0, mean velocity at channel entrance; $U = U_0 - Vx/h$, local mean velocity; $Re = Uh/\nu$, Reynolds number of main flow; $R = Vh/\nu$, injection (suction) Reynolds number; $\eta = y/h$, dimensionless coordinate, φ, amplitude of perturbations; α, wave number; c, speed of propagation of perturbations.

LITERATURE CITED

1. V. N. Varapaev and V. I. Yagodkin, "Flow stability in a channel with porous walls," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 5, 91 (1969).
2. V. M. Eroshenko, L. I. Zaichik, and V. B. Rabovskii, "Stability of fluid flow in a plane channel with uniform injection through porous walls," Inzh.-Fiz. Zh., 41, 436 (1981).
3. V. N. Varapaev and V. I. Yagodkin, "On the stability of certain nonparallel flows of a viscous incompressible liquid in a channel," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 4, 125 (1970).
4. A. Berman, "Laminar flow in channels with porous walls," J. Appl. Phys., 24, 1232 (1953).
5. L. L. Debruge and L. S. Han, "Heat transfer in a channel with a porous wall for turbine cooling application," Trans. ASME, J. Heat Transfer, 94, 385 (1972).
6. H. Schlichting, Boundary Layer Theory, McGraw-Hill, New York (1960).
7. V. I. Alimpiev, S. V. Kalinina, and P. P. Lugovskoi, "Investigation of hydrodynamics of flow of a single-phase medium in narrow gaps with injection," in: Turbulent Boundary Layer with Complex Boundary Conditions [in Russian], Novoskibirsk Inst. Teplofiz. (1977), p. 73.
8. M. A. Gol'dshtik and V. N. Shtern, Hydrodynamic Stability [in Russian], Nauka, Novosibirsk (1977).